The Comparison of Immunization Effects on the Treasury Bonds of China and the Us Based on Modified Duration

Qianhui Jin[1, *]

[1]Business Administration School, Zhongnan University of Economics and Law, Wuhan, Hubei Province, China

[*]Corresponding author: qhjin@gecedu.uu.me

Keywords: Immunization, Modified duration, Treasury bonds

Abstract: This paper takes the Chinese and American national debt market as the research object, uses the modified duration theory and immune strategy to construct different Chinese and American Treasury bond portfolios, calculates their respective yields, and then analyzes and compares the immunization effects of these portfolios by calculating the immunization error, and obtains the final conclusions: 1) the interest rate level and treasury bond maturity yields of the United States are lower than that of China, and the sensitivity of treasury bonds to interest rate risk is higher than that of China; 2) for the short durations under 10, the immunization errors of Chinese treasury bonds portfolio are bigger; 3) the immunization effects of long-term and short-term are different, which shows that the short-term interest rate disturbance has a certain reducing effect on the long-term interest rate.

1. Introduction

1.1 Duration

Duration is an important tool to measure the sensitivity of asset price to interest rate. From the mathematical point, after Taylor expansion of asset price at the equilibrium point, the first term is Macaulay duration, which is the derivative of price to interest rate [1].

$$\text{Macaulay duration}= \sum_{t=0}^{n} \text{t} \frac{\text{PV}(\text{CF}_t)}{\text{Bond price}} \tag{1}$$

Here, in formula (1), t -- time period (year)

$PV(CF_t)$-- present value of cash flows during year t.

In other words, Macaulay duration is the weighted average term to maturity of the cash flows from a bond. The weight of each cash flow is determined by dividing the present value of the cash flow by price.

The Modified duration measures the effect that 1% change in interest rates will have on the price of a bond or, say, a bond's interest rate risk by its price sensitivity.

$$\text{Modified duration}= -\frac{1}{P}\frac{\partial P}{\partial y} = \frac{1}{1+y}\sum_{t=0}^{n} t \frac{PV(CF)_t}{Bondprice} = \frac{1}{1+y} MacaulayDuration \tag{2}$$

Here, in formula (2), y represents "yield to maturity"(YTM).

On the basis of Macaulay duration, modified duration considers time value and is a dynamic revision of Macaulay duration.

1.2 Immunization Strategy

Immunization strategy is a method of interest rate risk management by combining bonds of different durations into a portfolio of a specific duration [2].

$$D_p = \sum_{i=1}^{n} W_i \cdot D_i \tag{3}$$

  DOI: 10.25236/iwass.2020.026

Here, in formula (3), D_v -- Macaulay duration of bond portfolio.

w_i -- the proportion of bond I in bond portfolio.

D_i -- Macaulay duration of bond I.

n -- number of bonds in bond portfolio.

By establishing an immunized bond portfolio, we can find a bond portfolio whose maturity is equal to the maturity of its liabilities (cash outflow). The core problem is to determine the proportion of various bonds in the bond portfolio.

1.3 Objective of Research

Under different circumstances of capital markets, the durations of the bonds at the same maturity are different because of the discrepancy between term structure and volatility of interest rate. The Treasury bonds of China and the US are exactly in two different capital market environments. The degree of interest rate marketization in China's capital market is relatively low, especially is the frequency of policy interest rate adjustment, and the policy interest rate is at a high level compared with that of the US. However, the capital market in the United States is developed, and the degree of interest rate marketization is relatively high. The policy interest rate is adjusted on a daily basis, and the policy interest rate is relatively low. When the volatility and level of interest rate are different, it is not known whether the effects of duration immunization strategy on bonds are different.

Therefore, in order to further explore the different characteristics of interest rates and how they will affect the effects of immunization strategy, this paper will compare the effects of immunization strategy based on the Treasury bonds of China and the US.

2. Review

G. O. Bierwag believed that as time passes, the duration of the initially chosen portfolio changes unless the portfolio is adjusted. An optimal dynamic adjustment rule for the immunizer is still the duration strategy [3]. More precisely, Yu and Huo noted that knowing that there exists negative correlation between price return and reinvestment return, it is proposed that choosing the right bond portfolio with the strategy of “Macaulay duration = holding period” can effectively avoid the risk of interest rate change [4].

Then how can we build up immunization models using duration? One method is to present a strategy of immunization that consists in matching duration and minimizing a new linear dispersion measure of immunization risk [5]. However, as Gloria M Soto stated, the empirical tests show that the success of duration-matching strategies is more primarily attributable to the number of risk factors considered rather than the particular model chosen [6].

Therefore, in this paper we tend to use more realistic data to find out the accurate effects of immunization strategies of treasury bond markets.

3. Methodology

3.1 Data

Firsr, we have collectes the data about treasury bonds of China and the US, which are necessary to calculate the modified durations. They are shown in Table 1 and Table 2 [7].

Table 1 Coupon Rates And Bond Prices of Treasury Bonds of China and the Us.

	China		US	
Maturity	Coupon rate (%)	Bond price (¥)	Coupon rate (%)	Bond price ($)
N=1	2.44	100.328	N/A	N/A
N=2	2.69	100.837	1.125	101.23
N=3	3.17	102.514	0.5	99.7
N=5	3.77	106.162	1.125	102.25
N=7	3.25	104.171	1.125	100.16
N=10	3.29	104.833	1.5	106.45

Table 2 Yield and Modified Duration of Treasury Bonds of China and the Us.

Maturity		1	2	3	5	7	10	15	20	30
CN	MD (year)	2.721097	5.433094	2.510040	4.033950	5.368428	10.83256	10.300783	12.748256	19.057382
	Yield (%)	2.083	2.254	2.32	2.443	2.622	2.693	3.045	3.14	3.227
US	MD (year)	N/A	3.713607	5.208333	8.346688	7.521738	15.245762	N/A	N/A	33.411678
	Yield (%)	0.403	0.496	0.6	0.664	0.795	0.822	N/A	N/A	1.411

3.2 Building Investment Portfolio

For a specific target duration, we choose two kinds of treasury bonds (same nation) whose maturity and modified duration are both the most close to that target duration, and calculate the ratio of the two. For example, when target duration is 3, we will choose 1-year and 5-year treasury bonds of China to build a portfolio. The reason why 2-year treasury bond is unavailable is that its modified duration is 5.433094, which is much much longer than 3. The complete results are showed in the Table 3.

Table 3 Portfolios Based On Duration Immunization Strategy (Year)

Target Duration		1	2	3	5	7	10	15	20	30
CN	Maturity A (ratio)	N/A	N/A	1 (0.79)	3 (0.13)	5 (0.56)	7 (0.37)	10 (0.49)	N/A	N/A
	Maturity B (ratio)	N/A	N/A	5 (0.21)	7 (0.87)	10 (0.44)	20 (0.63)	30 (0.51)	N/A	N/A
US	Maturity A (ratio)	N/A	N/A	N/A	2 (0.66)	3 (0.82)	7 (0.90)	7 (0.71)	10 (0.74)	10 (0.19)
	Maturity B (ratio)	N/A	N/A	N/A	7 (0.34)	10 (0.18)	30 (0.10)	30 (0.29)	30 (0.26)	30 (0.81)

Then we can work out the yield of each portfolio. Take China treasury bond portfolio with 3-year target duration as example. The process is as follows:

The minimum common multiple of 1 and 5 is 5, so the portfolio is: 79 yuan out of 100 yuan to buy 5 issues of one-year treasury bonds (i.e. reinvest 1 issue after one-year maturity, a total of 5 issues) and 21 yuan to buy 1 issue of five-year treasury bonds.

$$CF_0 = 0.79 \times \text{Bond Price}_1 + 0.21 \times \text{Bond Price}_5 = 0.79 \times 100.328 + 0.21 \times 106.162 = 101.55314$$

$$CF_1 = 0.79 \times 100 \times \text{Coupon rate}_1 + 0.21 \times 100 \times \text{Coupon rate}_5 = 79 \times 2.44\% + 21 \times 3.77\% = 2.7193$$

$$CF_1 = CF_2 = CF_3 = CF_4$$

$$CF_5 = 100 + 0.79 \times 100 \times \text{Coupon rate}_1 + 0.21 \times 100 \times \text{Coupon rate}_5 = 102.7193$$

$$\sum_{t=1}^{5} \frac{CF_t}{(1+y)^t} = CF_0$$

$\therefore$ y=0.428796

In this way, we can get the yields of all the above portfolios. The results are shown in Table 4.

Table 4 Yields of All Portfolios (%).

Target Duration		1	2	3	5	7	10	15	20	30
Yield	CN	N/A	N/A	0.428796	0.623559	0.337189	1.013324	0.714707	N/A	N/A
	US	N/A	N/A	N/A	0.430119	0.348184	0.771542	0.793829	0.210461	0.062994

3.3 Immunizzation Effects

Based on the yields of treasury bonds and yields of portfolios we have obtained, then we can easily work out the immunization error of each portfolio. The formula is:

$$\text{Immunization Error} = \frac{\text{Yield of portfolio(t)-Yield of treasury bond(t)}}{\text{Yield of treasury bond(t)}} \times 100\%$$

For example, when t=3, for the portfolio of China's treasury bonds whose target duration is 3, the immunization error of this portfolio equals:

$$\text{Immunization Error}=\frac{0.428796-2.32}{2.32}\times 100\%=-81.52\%$$

Similarly, all the immunization errors are worked out in Table 5.

Table 5 Effects of Immunization of Treasury Bonds of China and the Us (-%).

Target Duration		1	2	3	5	7	10	15	20	30
Immunization Error	CN	N/A	N/A	81.52	74.48	85.83	62.37	76.53	N/A	N/A
	US	N/A	N/A	N/A	35.22	56.20	6.14	N/A	N/A	95.54

It can be seen that for a short target duration (under 10), there is significant difference between effects of immunization of China and the US's treasury bonds. The most significant one is 7-year portfolios, the difference of which is 56.23%. However, for the longer target duration, we do not know exactly how these effects differ but the immunization effect of 30-year portfolio of the US would vary greatly compared to itself. This may be due to the fact that the longer the term is, the greater the deviation of risk premium is and the greater the volatility over time is, according to the decomposition of forward interest rate curve in treasury bond market based on excess return factor[8].

4. Conclusion

First of all, we compare the characteristics of treasury bonds of China and the US, and come to know that interest rate of the US is at a lower level for a long time and YTM of US treasury bonds is several percentage points lower than that of China. At the same time, the price of US Treasury bonds is also more sensitive to interest rates, even twice as much as that of China.

Based on the method of constructing immunization strategies in this paper, the effects of treasury bonds of China and the US are obviously different, especially for the short durations under 10. The immunization errors of China's treasury bonds portfolio are bigger. However, for the longer durations, we cannot conclude that confidently due to the lack of data, but we suppose that the immunization effects of long-term and short-term target durations will be quite different. According to Fama and Bliss's research, the forecast fuction of forward interest rate is weaker than that of spot interest rate, but it has a stronger explanation in the 4-to-5-year and other longer-term treasury bonds [9].

Let us do further analysis of the reasons for the differences between the durations, yields and immunization effects of China US Treasury bonds. By analyzing the calculation formula of duration, we know that the difference between the two bonds' parameters depends ultimately on the different coupon rates. The coupon rates of treasury bonds have always been the standard index of national interest rate. As the policy rate serves as a benchmark for the long-term interest rate, this paper also further explains the leverage effect of interest rate policy on capital market. In addition, the immunization effects of long-term and short-term target durations are different, which means that the disturbance of short-end interest rates might have a certain reduction effect at the long-end interest rates [10].

The limitation of this paper is that only two kinds of bonds are taken into a portfolio using immunization strategy. There do exist possibilities that the difference between the duration immunization strategies of China and US treasury bonds will be weakened when the bond categories in the portfolio increase.

References

[1] Richard A. Brealey, Stewart C. Myers, Franklin Allen, Principles of Corporate Finance, 10th ed., New York: McGraw-Hill/Irwin, 2011, pp.50-53.

[2] Zvi Bodie, Alex Kane, Alan J. Marcus, Investmens. 9th ed., New York: McGraw-Hill/Irwin,

2011, pp.528-531.

[3] G. O. Bierwag. “Immunization, Duration, and the Term Structure of Interest Rates”. Journal of Financial and Quantitative Analysis, vol.12, no.5, pp.725-742, 1977.

[4] Xin Yu, Chunguang Huo. “Analysis of Immunization Strategy of Bonds Based on Duration Model”. Business, no.11, pp.55, 2012.

[5] When can you immunize a bond portfolio?, [online] Available: ideas.repec.org.

[6] Gloria M Soto. “Duration Models and IRR Management: A question of dimensions?”. Journal of Banking & Finance, vol. 2004, no.28, pp.1089-1110, 2004.

[7] Weijie Yang. “Calculation and analysis of yield, duration and insurable value of bond holding period --Using Python as a tool”. Times Finance, vol. 2020, no.2, pp.80-82, 2020.

[8] Yang Dong, Rui Chou, Hao Wang. “Analysis and Forecast of China’s Government Bond Yield”. China Journal of Economics, vol 6, no.3, pp.24-43, 2019.

[9] Fama E. F., Bliss R. R. “The information in long-maturity forward rates”. The American Economic Review, vol.77, no.4, pp.680-692, 1987.

[10] Shengbao Chou, Xuebiao Wang, Junfang Guo. “Accurate decomposition of forward interest rate curve based on prediction factors of excess return”. Statistics & Decision, vol.2015, no.7, pp.66-68, 2015.